By using (4) and the definition of $\underline{b}$ in (5b), this can be written as

$$W=\frac{1}{2}\left[\underline{V}_p^T\underline{V}_f^T\right]\begin{bmatrix}\underline{\underline{S}}_{pp}\underline{V}_p\\ -\underline{b}\end{bmatrix}. \quad (7)$$

Let us now assume that we have two conductors; one conductor has d nodes at potential v_0, the second conductor is at zero volts. Therefore, we have

$$\underline{V}_p=[V_0V_0\cdots V_0 00\cdots 0]^T.$$

Equation (7) reduces to

$$W=\frac{1}{2}\left[\underline{V}_0^T\underline{V}_f^T\right]\begin{bmatrix}\underline{\underline{S}}'_{pp}\underline{V}_0\\ -\underline{b}'\end{bmatrix} \quad (8)$$

where $\underline{V}_0$ has dimension d and $\underline{V}_0 \triangleq [V_0V_0\cdots V_0]$ and $\underline{b}' \triangleq -\underline{\underline{S}}'_{fp}\underline{V}_0$.

The primes on $\underline{\underline{S}}$ and $\underline{b}$ indicate that the dimension has been reduced and will be dropped henceforth. From the zero-row-sum property of $\underline{\underline{S}}$

$$\underline{\underline{S}}_{pp}\underline{V}_0=-\underline{\underline{S}}_{pf}\underline{V}_{00}. \quad (9)$$

The dimension of $\underline{V}_{00}$ is equal to the number of free unrestrained nodes, viz

$$\underline{V}_{00}=[V_0V_0\cdots V_0].$$

Again, using the fact that the quadratic form is equal to its own transpose

$$\underline{V}_0^T\underline{\underline{S}}_{pf}\underline{V}_{00}=\underline{V}_{00}^T\underline{\underline{S}}_{fp}\underline{V}_0. \quad (10)$$

By substituting (9) and (10) into (8), we obtain

$$W=\frac{1}{2}\left[\underline{V}_{00}\underline{V}_f\right]\begin{bmatrix}\underline{\underline{S}}_{fp}\underline{V}_0\\ -\underline{b}\end{bmatrix}=\frac{1}{2}\left[\underline{V}_{00}^T\underline{V}_f^T\right]\begin{bmatrix}\underline{b}\\ -\underline{b}\end{bmatrix}$$

$$W=\frac{1}{2}\left(\underline{V}_{00}^T-\underline{V}_f^T\right)\underline{b}. \quad (11)$$

Conclusion

Three equations useful in calculating the energy, from which capacitance can be found, can be used. These are (3), (6), and (11). Equation (11) is the same as that reported in Daly and Helps' letter [1] (also eq. (11) in that paper), but which we arrived at by treating energy on a global basis. It is useful because the $\underline{b}$ vector is presumably available in the process of solving (5b).

We have been using (6) to extract energy from our own finite-element program. The advantage here is that only elements which border on restrained boundaries need to be considered, the other contributions being zero. Equation (6) is also more general than (11), since it is not constrained to two conductors, one at V_0 V and the other at zero volts.

Which of the above equations is most useful depends on how the global stiffness matrix is solved and the specific geometry of the problem involved.

References

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Reply[1] by P. Daly[2]

Kaires and Beren have derived three equations for calculating the energy stored in an electrostatic structure by various manipulations of the governing variational expression as discretized in finite-element form. One of these equations originally derived in 1972 by Daly and Helps (reference [1] of their letter) allows the direct computation of capacitance by a simple matrix operation.

Kaires and Beren's three equations (presumably equivalent taking boundary elements into account) would have additional force if the authors had indicated the area of application of each. The last line of their conclusions gives no real guidance on this important question to the reader.

In the abstract, Kaires and Beren state that our energy minimization is carried out on an element-by-element basis. We concede that the wording of our original letter might give such an impression but it is obvious that, in fact, any minimization is and must be global. This point is reinforced by the fact that Kaires and Beren themselves reproduce our (11) by minimizing globally!

The authors are incorrect in stating that our equation for capacitance is constrained to two conductors, one at V_0 volts and the other at zero volts. In fact, our equation allows any potential difference ϕ between conductors: this is demonstrated by its ability to handle [4] odd modes (positive, negative, and zero potentials) in coupled transmission lines.

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Correction to "A Continuous Comparison Radiometer at 97 GHz"

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In the above paper,[1] the reference to Faris [1] should not imply that the correlation radiometer was invented by Faris. This class of radiometers was devised by Dr. Emil Blum and described in his paper "Sensibilite des radiotelescopes et recepteurs a correlation," which appeared in *Annales D'Astrophysique*, vol. 22, no. 2, pp. 139–163, Mar.–Apr. 1959. Dr. Blum's contribution was also cited in the paper "Radio telescopes," published in *Methods of Experimental Physics*, vol. 12, pt. B, pp. 218–219.

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